UNCLASSIFIED

K-736

Subject Category: PHYSICS

UNITED STATES ATOMIC ENERGY COMMISSION

THE ENERGY SPECTRUM OF LEAKAGE NEUTRONS FROM A HOMOGENEOUS REACTOR

By
R. C. Rohr
H. F. Henry

[redacted]

CONTRACT NO. W-7405-ENG-26

July 5, 1951

K-25 Plant
Carbide and Carbon Chemicals Company
Oak Ridge, Tennessee

Technical Information Service, Oak Ridge, Tennessee

UNCLASSIFIED

Date Declassified: December 27, 1955.

Issuance of this document does not constitute authority for declassification of classified material of the same or similar content and title by the same authors.

Printed in USA, Price 25 cents. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

Date of Issue: July 5, 1951 Report Number: K-736

THE ENERGY SPECTRUM OF LEAKAGE NEUTRONS FROM A HOMOGENEOUS REACTOR

R. C. Rohr and H. F. Henry

Safety and Inspection Division
K. W. Bahler, Superintendent

ABSTRACT

The leakage neutron spectrum from a chain reaction in an aqueous solution of uranyl nitrate has been determined for the purpose of calibrating indium foil to be used for personnel monitoring of a possible accidental critical reaction at K-25. Approximately 75% of the neutrons were found to be slow, and experimental results are in reasonable agreement with a semi-theoretical treatment of the problem. The overall cross section of indium for neutrons of all energies in this leakage spectrum was found to be about 60 ± 4 barns. This is of the same order of magnitude as the value of 126 barns which was determined from the calculated energy spectrum using weighted averages of directly measured cross sections.

CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION
K-25 Plant
Oak Ridge, Tennessee

Work performed under Contract No. W-7405-Eng-26.

THE ENERGY SPECTRUM OF THE LEAKAGE NEUTRONS FROM A HOMOGENEOUS REACTOR

TABLE OF CONTENTS

THE ENERGY SPECTRUM OF THE LEAKAGE NEUTRONS FROM A HOMOGENEOUS REACTOR

INTRODUCTION

The K-25 Special Hazards Committee has specified that indium foils be placed in all personnel security badges for the purpose of monitoring an individual's exposure to a possible accidental critical reaction.[1] It thus became desirable to calibrate such foils by use of a water-moderated fission neutron source such as might be anticipated at K-25. Accordingly, the leakage neutron energy spectrum from such a fission source has been determined. No information on such a spectrum has been found even though the energy spectrum of the uranium-235 fission neutrons has been experimentally measured by several workers.[2,3] The following basic assumptions have been made:

1. Neutron slowing down data for water may be used to determine the energy of fission neutrons which escape the reactor.
2. Both the inelastic scattering due to the solute and the thermal neutron absorption of the water and the solute are negligible.
3. Fissions occur homogeneously throughout the reactor.
4. The slowing down effects of water and paraffin are the same.
5. The presence of phosphorus and nitrogen in the reactor fuel have no effect on the neutron spectrum.
6. The neutron counters have the same uniform sensitivity[4] for the energy range from thermal to values above 5 Mev.
7. All activations of foils covered by cadmium are due to neutrons of indium resonance energy.

SUMMARY OF RESULTS

An experimental leakage spectrum of a moderated but unreflected fission neutron source was determined by measuring the indium resonance neutron flux at differing thicknesses of paraffin, and from these measurements determining the original energy of the neutrons which had been moderated in each given paraffin thickness to the indium resonance energy. This spectrum is given in figure 1, and the fraction of neutrons in certain energy intervals is given in table V. These fractions are compared with similar values given by a semi-theoretical spectrum which was obtained by determining the slowing down effect of the water on the neutrons emitted by fission throughout the reactor and evaluating the resultant leakage spectrum from the modification thus produced in the measured uranium-235 fission spectrum.

---

1 Visner, S., Minutes of Meeting of K-25 Special Hazards Committee, June 8, 1949 (KS-53).

2 See for example Perlman, I. H., and Richards, H. T., The Fission Spectrum of 25, February 11, 1944 (LA-60)

3 Watt, B. E., Energy Spectrum of Neutrons from Fissions Induced by Thermal Neutrons, December 17, 1948 (LA-718)

4 Hanson, A. O., and McKibben, J. L., A Neutron Detector Having Uniform Sensitivity from 10 Kev to 3 Mev, February 11, 1947 (MDDC-972)(LADC-409)

It should be noted that experimentally approximately 90% of the neutrons were found to be thermal, but that the semi-theoretical treatment indicated this fraction was approximately 65%.

From the experimental data, an effective indium cross section of 60 ± 4 barns[5] was found for this spectrum. This figure is in reasonable agreement with the 126 barn value calculated from the cross sections given in the literature[6] and the spectrum determined semi-theoretically.

DESCRIPTION OF EXPERIMENT

Apparatus

Most of the apparatus used has been completely described in other reports.[7] The solution used in these experiments was an aqueous solution of uranyl nitrate with phosphoric acid impurity, the choice of which was purely a matter of availability for a single experiment. The experimental work was performed at an H/U-235 of 316, a P/U-235 of 53, and an N/U-235 of 2.9 where the symbols stand for atomic ratios of hydrogen, phosphorus, and nitrogen to uranium-235, respectively.

Most of the indium foils used for the experiment were 20 x 20 x 0.2 mm. No effort was made to prepare foils of uniform size and thickness, but all foils were weighed and the data plotted in units of dis./min./g.

Foils were beta counted with an end window type GM counter having a mica window. This counter was mounted inside a lead "pig" which reduced background counts to about 23 counts/min. and was connected to a Nuclear Instruments Model 164 scaler.

Method

One boron-lined neutron counter was calibrated with a polonium-beryllium source of known strength so that the neutron flux across the surface of the counter would be known. This counter was then used as a standard for the calibration of other instruments which could be used for high flux measurements.

The procedure followed in the experiment was first to bring the reactor system to criticality as previously described.[7] Then, with safety rods in and the system thus subcritical, foils were placed at 142 cm. from the reactor with sets of 2 foils, 1 bare and 1 inside a cadmium sandwich, being placed between various thicknesses of paraffin. The system was then returned to criticality and the power raised to the desired level which required about 2 minutes. Foils were exposed for 108 minutes after which the safety rods were again inserted and the foils removed. Since the neutron flux fell to zero almost instantly, decay time was counted from the moment of insertion of the safety rods. No precautions were taken to protect the foils from scattered neutrons, since the experiment was designed to include effects due to scattered neutrons.

---

5 See appendix C for the derivation of this equation

6 Way, K., and Haines, G., Tables of Neutron Cross Sections, October 28, 1947 (MonP-405)

7 Beck, C. K., Callihan, A. D., Morfitt, J. W., and Murray, R. L., Critical Mass Studies, Part III, April 19, 1949 (K-343)

Each foil was allowed to decay at least 10 minutes after exposure to allow the 13 and 72 second activities to become negligible, and all foils were counted within 2 hours after the time of exposure. Foils were counted on one side only since no significant differences were found with the foils which were counted on both sides. Since the 54 minute half-life accounted for more than 99% of the activity observed from several measurements of the indium foil activities, no other half-life has been considered. The data from which the energy spectrum was determined empirically are given in table I and the results are plotted in figure 1.

TABLE I

INDIUM FOIL PARAFFIN DATA
(Leakage Neutron Flux = 3.435×10^4 neutrons/cm.^{2}/sec.
142 cm. from the reactor core)
Exposure Time = 108 Minutes

Thickness of Paraffin (cm.)	Average Energy	Energy Range	Counts/ Minute	Decay Time (min.)	Foil Weight m(g.)	Saturated Activity A_s (dis./ min./g.)	Relative Flux/ev. (Arbitrary Units)
0	Th.&1.44 ev.	---	594*	77	0.784	37,900	---
---	1.44 ev.	0.20 ev.	254	75	0.786	15,800	7.9×10^4
---	Thermal	0.40 ev.	---	--	---	22,100	6.6×10^5
0	Th.&1.44 ev.	---	803*	93	1.61**	30,600	---
---	1.44 ev.	0.20 ev.	489	41	1.75**	8,870	4.4×10^4
---	Thermal	0.40 ev.	---	--	---	21,800	6.5×10^5
3.8	9 ev.	2.0 ev.	210	59	0.860	9,750	4.9×10^3
7.6	0.15 Mev.	0.042 Mev.	136	69	0.850	7,280	0.173
11.9	1.28 Mev.	0.034 Mev.	144	54	0.794	6,880	0.202
15.2	2.70 Mev.	0.036 Mev.	147	72	0.851	8,060	0.224
19.1	4.60 Mev.	0.032 Mev.	140	48	1.73**	5,680	0.177

* Bare foils; all others were cadmium covered.
** These foils were 25 x 38 x 0.2 mm.; all others were 20 x 20 x 0.2 mm.

The combined geometry and efficiency of the GM counter used to determine foil activations was measured with a 1 cm.^{2} radium D + E standard source, NBS Number 3171, in the same counting geometry as that used for the foils and was found to be 0.242. Since the energies of the radium beta rays are approximately the same as the indium beta ray energies, no discrepancy in overall counting efficiencies due to energy differences should occur although slight inaccuracies were introduced by the size of the foils whose areas were greater than the 1 cm.^{2} area of the radium source.

The standard radium source may be assumed to have no self-absorption, but the indium foils will have a considerable amount of self-absorption. Thus, in cases where an absolute disintegration rate is needed, it is necessary to determine the self-absorption in the indium foils. This was obtained empirically by exposing a set of foils to the standard σ pile at X-10 at a position for which the thermal flux is well known. From the known thermal neutron flux and the thermal neutron absorption cross section for indium, the theoretical saturated activity was calculated. This was compared with the saturated activity of the exposed foils which had not been corrected for self-absorption and the ratio of the two gives a correction factor of 0.150 ± 0.021. Thus, in all tables, the apparent saturated activity determined directly from the counting rate was divided by a factor of 0.0363 to correct for the counting geometry and self-absorption in determining the true saturated activity.

Experimental Determination

It is assumed that the beta activity of the foils is a measure of the neutron flux which strikes them. Preliminary data indicated that much of this activity was due to thermal and low energy neutrons below the cadmium cutoff at 0.4 ev. since cadmium covers over the foils greatly reduce their activity as indicated by the data in table II. Thus, the difference between bare foil activations and cadmium covered foil activations will be a function of the low energy neutron flux, identified in the remainder of this paper as thermal neutron flux. Activations of foils in cadmium sandwiches are considered to be a function of the resonance energy neutron flux only.

TABLE II

EFFECT OF CADMIUM FILTERS

Condition	Saturated Activity	Condition	Saturated Activity
1A *	66493	5A	81107
1B **	23832	5B	31636
2A	149588	6A	187627
2B	71036	6B	97042
3A	401747	7A	623065
3B	256281	7B	337682
4A	617228	8A	899443
4B	344806	8B	475578

*A indicates bare foils.
**B indicates foils in cadmium sandwich.

The experimental data given in table III indicate that when a foil has several inches of paraffin behind it, 50% of its thermal neutron activity may be considered to be due to direct neutrons and 50% due to back scattered neutrons. These results were determined by bare foil comparisons both with cadmium sandwich foils which had 0.051 cm. cadmium filters on both sides of the indium foil, and with cadmium faced foils which had the 0.051 cm. cadmium only on the side nearest the reactor. All foils described in table III have 19 cm. of paraffin behind them. The data were taken during preliminary experiments in which the power level of the reactor and foil activities were very low so that the statistics are rather poor. The results, however, are considered to be indicative. For this reason, only half of the activities of the foils placed between paraffin barriers as given in table I were used to evaluate the neutron flux.

TABLE III

NEUTRONS BACK SCATTERED BY PARAFFIN

Filter	Saturated Activity *	Thermal Neutron Activity	% Due to Back Scattered Neutrons
Cadmium Sandwich	6910	---	---
Cadmium Faced	10638	3728	---
Bare	15301	8391	44.4
Cadmium Sandwich	3428	---	---
Cadmium Faced	9938	6510	---
Bare	13607	10179	64.0
Cadmium Sandwich	600	---	---
Cadmium Faced	1194	594	---
Bare	1794	1194	49.7
Cadmium Sandwich	1681	---	---
Cadmium Faced	4389	2708	---
Bare	8884	7203	37.6
		Av.	48.9

*The term saturated activity as used in the data is defined as the activity which the foil would have immediately after an exposure to neutrons for an infinite time. The equation for the saturated activity derived in appendix B is a function of the neutron flux, decay constant, and times of activation, decay and counting.

The neutron fluxes at the higher energies were calculated from Nordheim's slowing down data,[8] which include the effects of neutron absorption, and from the measured cadmium-covered foil activations produced by the neutrons moderated by the different thicknesses of paraffin. It was assumed that the only neutrons that activate these foils are those which are slowed to the resonance capture energy of the paraffin. Although this indium neutron capture resonance at 1.44 ev. is sharply peaked, it has a width of approximately 0.2 ev.; hence, it was assumed that the activation produced in any foil was actually produced by neutrons in the energy range of 1.34 - 1.54 ev. Thus, the foil activation produced by neutrons passing through a given thickness of paraffin was assumed to be proportional to the total incident neutron flux in an energy range which would be slowed to energies between 1.34 and 1.54 ev. by the paraffin. Obviously, this energy range of the incident neutrons will depend upon the average incident neutron energy. The actual energy ranges corresponding to the slowing down produced by differing

---

[8] Nordheim, L. W., Nordheim, G., and Soodak, H., Slowing Down of Neutrons in H_2O and in Graphite, January 20, 1949 (CP-1251)(A-1827)

thicknesses of paraffin were determined graphically from Nordheim's data[9] and listed in table I.* Except for thermal neutrons the points plotted in figure 1 were obtained by dividing the foil activations by the energy range concerned in each case.

The thermal neutron flux was determined from the difference in the activations of bare foils and cadmium-covered foils placed on the front face of the paraffin. This difference was assumed to be due to the foil activation produced by neutrons of energies below 0.4 ev. which is the cadmium cutoff energy. The remainder of the bare foil activation, which was that measured by the cadmium-covered foils, was considered as being in the energy range of 0.4 - 2.0 ev. since the neutron capture cross section of indium is small for energies above 2.0 ev. The average indium cross sections for these two energy ranges were determined graphically, and the results were used with the activation data to evaluate the ratio of the thermal and indium resonance fluxes. The value of 12 obtained for the ratio of the average resonance to thermal neutron cross sections for indium is in reasonable agreement with the value of 13.9 used by Sargent and Duckworth.[10]

The experimental data for the leakage spectrum were plotted on standard coordinate paper and graphically integrated to obtain the percentage of slow neutrons defined as those in the energy range of 0 - 1000 ev. The result of this calculation indicated that 90% of the neutrons are in this range.

THEORETICAL CONSIDERATIONS

Energy Spectrum

The energy spectrum of the neutrons which escape the reactor will be a function both of their initial energy and of the solution path through which they travel because the hydrogen content of water has such a large slowing down effect upon the neutrons. Using the known fission neutron energy spectrum[2,3], a semi-theoretical determination of the energy distribution of the leakage neutrons was made for a 15 in. diameter cylindrical reactor using a uranium compound in an aqueous solution.

In this method, the locus of all points such that the thickness of solution, indicated as $\underline{m}$ in figure 2, is constant between a point A in the solution and the detector point, P, was first determined. The problem is rather simple for 2 dimensions, but becomes more complicated with an extension to 3 dimensions. For this reason, the variation in $\underline{m}$ with the height of the fuel was neglected and an average value was used. The possible error introduced by this assumption is not serious for large distances between the detector and the reactor.

As shown in appendix A, the length of the arc so defined is given by the relation:

$$L = 2\left\{ m\,\Theta + d\sin\Theta - \frac{a^2}{d}\left[F\,(\beta,\alpha) - \frac{1}{Q^2}\Big(F\,(\beta,\alpha) - E\,(\beta,\alpha)\Big)\right]\right\}$$

9 Nordheim, L. W., Nordheim, G., and Soodak, H., op. cit.

* The slowing down distances were taken to be the square root of the second moment of the slowing down range $\overline{r^2}$, Figure 1 of Report CP-1251.

10 Sargent, B. W., and Duckworth, J. C., Cadmium Ratios with Thin and Thick Indium Foils in the NRX Pile and in Other Lattices, September 12, 1950 (CRP-459) (PD-242)

where

$\textcircled{H}$ = half angle intercepted by the reactor at the detector

a = radius of the reactor

F,E = elliptic integrals which have been tabulated[11]

m = thickness of solution between the arc and the detector

d = distance from the center of the reactor to the detector point

$Q = \frac{a}{d}$.

The source volume of a group of neutrons was taken as the product of the length of arc defined above, the unit depth, and the solution height. The fission spectrum of the neutrons produced in such a volume was broken into 8 energy ranges as shown in table IV, and the water moderation[12] produced by a given thickness of solution, m, was calculated for each range. Since each range of energies emerging from the cylinder after moderation has a new energy value, the spectrum of the leakage neutrons was determined from these values and the fraction of the total volume in which they originated. The fission spectrum range between thermal energies and 6 Mev. was used, since less than 2% of the fission neutrons[2,3] have energies above 6 Mev. and no slowing down data for neutrons of higher energies were available. The resulting spectrum was found to be a function of the position of the detector as shown by table IV with the percentage of thermal neutrons being 60 - 75% of the total for the various points calculated. It is thus obvious that the calculated energy spectrum will depend slightly upon the distance of the detector from the reactor.

TABLE IV

NEUTRON ENERGY DISTRIBUTION (%)
(CALCULATED)

Energy* (Mev.)	6	5	4	3	2.5	1.75	0.75	0.25	Slow
Energy Interval (Mev.)	7.0- -5.5	5.5- -4.5	4.5- -3.5	3.50- -2.75	2.75- -2.00	2.00- -1.50	1.5- -0.5	0.50- -0.001	0- -0.001
dist. d,(cm.)									
25	0.35	0.83	0.91	1.13	2.41	4.13	7.98	9.33	72.89
35	0.45	1.09	1.08	1.69	2.93	4.97	9.43	10.00	68.34
84	0.55	1.20	1.35	2.01	3.49	6.00	8.26	13.72	63.45
142	0.59	1.35	1.38	2.16	3.75	6.43	11.46	11.74	61.16
Average	0.49	1.12	1.18	1.75	3.15	5.38	9.28	11.20	66.46

*All neutrons in the energy interval concerned were treated as through they had this energy.

11 Jahnke, E., and Emde, F., Tables of Functions, Dover (1945). The functions are defined in equations 7, 8, and 9 of appendix A.

12 The water moderation was taken from curves given in the report by Nordheim, L. W., Nordheim, G., and Soodak, H., op. cit.

Comparison of Theory and Experiment

In order to compare the experimental spectrum with the calculated flux distribution, it was necessary to integrate the results over comparable ranges. The values shown in table V are the results of such integration for ranges whose approximate midpoints are the experimental energies.

TABLE V

LEAKAGE NEUTRON DISTRIBUTION

Energy	Energy Interval (Mev.)	Per Cent of Neutrons in Interval Experimental	Calculated
Slow	0 - 0.001	89.9	66.46
0.150 Mev.	0.001 - 0.500	4.9	11.20
1.28 Mev.	0.500 - 2.00	3.4	14.66
2.70 Mev.	2.00 - 3.50	0.7	4.90
4.60 Mev.	3.50 - 7.00	1.0	2.79

EFFECTIVE CROSS SECTION

Using the fundamental relation of neutron activations together with the experimental data obtained, the following expression[13] for the effective, or average, indium cross section for the leakage neutrons, $\sigma_{eff.}$, was obtained:

$$\sigma_{eff.} = \frac{a\,A_s}{k\,n_o}$$

where

a = target area = area of foil

A_s = saturated activity of the foil [14]

k = number of target atoms/g. = $\frac{\text{Avogadro's number}}{\text{atomic weight}}$

(k = 5.25×10^{21} for indium)

n_o = number of incident neutrons/min.

From the absolute disintegration rates of the 20 bare foils, not backed by paraffin, used in the experiment, the average value of the effective cross section, within the 95% confidence belt, was found by this method to be

$$\sigma_{eff.} = 60 \pm 4 \text{ barns.}$$

13 See appendix C for the derivation

14 Calculated from formula 11 of appendix B using the constants given there.

The data used in this determination are given in table VI. The neutron fluxes were determined from data obtained with a neutron counter calibrated with a polonium-beryllium source of known strength as previously described.

TABLE VI

EFFECTIVE CROSS SECTION DATA

Distance from Reactor Core(cm.)	Counts (C)	Count Time (τ) (min.)	Decay Time (T) (min.)	Foil Weight (m) (g.)	Saturated Activity(A_s) x 10^{-5} (dis./min./g.)	Flux x 10^{-5} n./cm.^{2}/min.	Eff. Cross Section ($\sigma_{eff.}$) (barns)
142	2306	2	25	1.579	0.373	1.26	56.4
142	2437	2	28	*	0.395	1.26	59.6
86	4437	2	35	1.593	0.811	2.70	57.2
86	4122	2	30	*	0.685	2.70	48.3
36	9408	2	43	1.620	1.88	6.20 **	57.7
36	19294	3	40	*	2.45	6.20 **	75.3
25	15389	1	48	1.685	6.25	17.6	67.7
25	15043	1	50	*	6.43	17.6	69.6
19	19180	1	55	1.597	8.99	27.3	62.7
19	19644	1	54	*	8.83	27.3	61.6
142	803	2	93	1.608	0.293	0.94	59.4
142	1016	2	63	*	0.258	0.94	52.2
86	1633	2	98	1.603	0.666	2.02	62.9
86	1691	2	96	*	0.656	2.02	61.9
36	3224	2	104	1.593	1.43	4.63**	58.8
36	2425	2	107	*	1.08	4.63**	44.6
25	8864	2	112	1.726	4.02	13.1	58.4
25	9963	2	114	*	4.87	13.1	70.8
19	11951	2	119	1.658	6.17	20.4	57.7
19	12687	2	122	*	6.87	20.4	64.2
						Av.	60 ± 4

*The weights for these foils were not obtained; hence, the average weight (1.644 g.) of all other foils, some of which were not used in this determination, was used in the calculations.

**The fluxes for these foils were multiplied by 0.59 to correct for the fact that the activity of the foils falls so far below the straight line of figure 3.

A graph of foil activation as a function of distance from the center of the reactor as shown in figure 3 indicates that, empirically, the foil activities are inversely proportional to the 3/2 power of the distance. If it be assumed that the foil activation is directly proportional to the neutron flux, then the neutron flux likewise varies inversely with the 3/2 power of the distance.

The actual neutron flux was determined for a point at a distance of 142 cm. by use of a calibrated counter as previously stated, and the fluxes for the other detector points of figure 3 were determined from this value and the above flux-distance relation.

It will be noted in figure 3 that the activities of the foils which were at a distance of 36 cm. from the source and which were in contact with the outside of the 1/8 in. steel tamper tank differ markedly from the values indicated by the activation-distance relation. It should be emphasized, however, that this relation is purely empirical and may even be fortuitous. Thus, no effort has been made to correlate it with theory or to account for the deviation at 36 cm. Accordingly, in determining the data of table VI which were used to evaluate the average indium cross section, the fluxes calculated at the 36 cm. point by the above method were multiplied by a correction factor of 0.59 so that they would be proportional to the measured activations at that point.

This 60-barn experimental cross section value may be compared with a weighted average cross section of 126 barns determined from published cross section values and the semi-theoretical energy spectrum indicated in table IV.

Although the experimental value is obviously low in comparison with the weighted average cross section, it is of the correct order of magnitude, which is the best that might be expected considering the inaccuracies inherent in the method indicated for the determination of the absolute neutron flux and those involved in the evaluation of an absolute beta disintegration rate. It should be noted that the literature value for the indium thermal neutron cross section is 190 barns.[15]

DISCUSSION OF ERRORS

In the semi-theoretical determination of the neutron flux, the simplifying assumption was made that the fission activity of the reactor was uniformly distributed throughout its volume. It is obvious that if this assumption is not true, the energy distribution of the neutrons escaping from that reactor will be different from that actually calculated. For an unreflected reactor, the neutron flux at its center is higher than it is at the edges. If this flux density correction were applied to the calculations, it is obvious that the fraction of neutrons in the higher energy ranges would be reduced. In this case, the agreement between the theoretical and experimental data would be improved.

Similarly, two other assumptions made in the calculations have the effect of indicating too many high energy neutrons. Both are concerned with corrections whose importance decreases as the distance between the reactor and the detector is increased. One involves the method of use of a constant value for the water thickness, $\underline{m}$, which was independent of the cylinder height, while the other involves a simplification in the determination of the arc length, $\underline{L}$, bounding a zone in the calculations.

The experimental data are subject to several sources of error which are believed to be small but which will affect the results. Of these, probably the largest single one is the assumption that the cadmium filters remove all

15 Way, K., and Haines, G., op. cit.

thermal neutrons. Although calculations show that about 98% of the thermal neutrons are removed by the filters, it is apparent that if the flux is overwhelmingly thermal, these thermal neutrons may produce most of the activation which is thus incorrectly attributed to resonance energy neutrons. For foils behind several inches of paraffin, the incident flux may be very largely thermal, and thus the flux indicated on the graph at the higher energies is probably too great. The cadmium filters were such that the cadmium overlapped the indium by at least 1/4 in. on all sides, and precautions were taken to keep them in contact so that the foils were protected from thermal neutrons scattered from the edges of the cadmium sandwich.

Dead time corrections were not made. Since the counts are of the order of 500 counts per minute, dead time corrections would be negligible.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the assistance of Dr. A. D. Callihan in helping organize and perform the experiment, the very helpful discussions in the planning of the experiment which were held with him and with Messrs. J. W. Morfitt, D. F. Cronin, and J. K. Fox, the review of the manuscript by Dr. R. L. Macklin, and the contributions to the evaluation of the data by Mr. J. C. Bailey.

APPENDIX A

EQUATION FOR LENGTH OF ARC

L = Arc Length = $\int_c ds$

From figure 2,

$$a^2 = d^2 + \rho^2 - 2\rho d \cos\theta$$

$$m = r - \rho$$

1) $r = m + d\cos\theta - \left[a^2 - d^2 \sin^2\theta\right]^{1/2}$

2) $b^2 = a^2 + m^2 - 2\,am\cos\phi$

θ and ϕ will reach limiting values Θ and Φ respectively, when $b = a$ (figure 2) and thus

3) $m - 2\,a\cos\Phi = 0$

From figure 2,

4) $d\sin\Theta = a\sin\Phi$

From equations 3) and 4) and the trigonometric identity $\sin^2\Phi + \cos^2\Phi = 1$

$$1 = \frac{m^2}{4a^2} + \frac{d^2}{a^2}\sin^2\Theta$$

$$\therefore \Theta = \sin^{-1}\frac{1}{2d}\left(4a^2 - m^2\right)^{1/2}$$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

The integral in its complete form becomes very complicated so that a simplification was used.

Let $r_\theta = \frac{dr}{d\theta}$

$$\left(r_\theta^2 + r^2\right)^{1/2} = r\left[1 + \frac{r_\theta^2}{r^2}\right]^{1/2}$$

$$= r\left[1 + 1/2\,\frac{r_\theta^2}{r^2} - 1/8\,\frac{r_\theta^4}{r^4} + 1/16\,\frac{r_\theta^6}{r^6} - \ldots\right]$$

The series is obviously convergent for values of $r_\theta \leq r$, and the maximum error of such a convergent series will be less than the error for the first term neglected. For the range over which the integration is made in this case, $\frac{r_\theta}{r} < 1$ and a numerical analysis of this error for the nearest detector point, in first order correction, showed it to be between 0 and 23% with an average of less than 12%. Since the experimental errors are probably of this order of magnitude and since the complete expression is rather complicated, only the zero order term has been considered.

Thus,

$$5)\quad L = 2 \int_0^{Ⓗ} r\, d\theta$$

$$= 2 \left\{ m Ⓗ + d \sin Ⓗ - \int_0^{Ⓗ} \left[a^2 - d^2 \sin^2\theta\right]^{1/2} d\theta \right\}$$

By a change of variables the expression obtained for the integral term is

$$\int_0^{Ⓗ} \left[a^2 - d^2 \sin^2\theta\right]^{1/2} d\theta = \frac{a^2}{d}\left[\int_0^{\beta} \frac{dx}{(1-x^2)(1-Q^2x^2)^{1/2}} - \int_0^{\beta} \frac{x^2dx}{(1-x^2)(1-Q^2x^2)^{1/2}}\right]$$

$$6)\quad \text{where } x = \frac{d}{a}\sin\theta,\ Q = \frac{a}{d} \leq 1$$

The integration is now in the form of two elliptic integrals, the formulae for which have been taken from Jahnke and Emde.[16]

$$7)\quad \int_0^{\beta} \frac{dx}{(1-x^2)(1-Q^2x^2)^{1/2}} = F\,(Q,\beta) = \int_0^{\Psi} \frac{d\psi}{(1-Q^2\sin^2\psi)^{1/2}}$$

where

$x = \sin\psi$

Ψ = limiting value of ψ

β = limiting value of x.

$$8)\quad \int_0^{\beta} \frac{x^2dx}{(1-x^2)(1-Q^2x^2)^{1/2}} = \int_0^{\Psi} \frac{\sin^2\psi\, d\psi}{(1-Q^2\sin^2\psi)^{1/2}}$$

$$= D\,(Q,\beta)$$

where

$$9)\quad D(Q,\beta) = \frac{F - E}{Q^2}$$

16 Jahnke, E., and Emde, F., Tables of Functions, Dover (1945) pp. 52 and 56

and

$$E(Q,\beta) = \int_0^{\psi} d\,\psi \left[1 - Q^2 \sin^2\psi\right]^{1/2}$$

$$10) \quad \therefore \quad L = 2\left\{m\,Ⓗ + d\sin Ⓗ - \frac{a^2}{d}\left[F - \frac{1}{Q^2}\,(F - E)\right]\right\}$$

APPENDIX B

SATURATED ACTIVITY

The rate of change of the number of radioactive atoms present in a foil during irradiation is dependent upon the rate of production and the normal decay. Assuming that the rate of production is constant and activities due to only 1 half-life are present

$$\frac{dN'}{dt} = n - \lambda N'$$

where

λ = decay constant in min.

n = number of atoms activated/min.

= number of neutrons captured/min.

N' = number of activated atoms at time t

t = exposure time in min.

Solving this equation, assuming there are no radioactive atoms at $t = 0$,

1) $N'\lambda = n\ (1 - e^{-\lambda t})$

After an infinite time, the rate of production will equal the rate of decay and

$$\frac{dN'}{dt} = o.$$

then

2) $n = N_s\lambda,$

where N_s = number of radioactive atoms in the foil at saturation. The term saturated activity, A_s, is defined as the number of atoms decaying/min. per gram at saturation.

Thus

$$A_s = \frac{n}{m} = \frac{N_s\lambda}{m}$$

$$\therefore\ N'\lambda = \frac{N_s\lambda}{m}\ (1 - e^{-\lambda t})$$

After the irradiation has ceased

3) $\frac{dN}{dT} = - N\lambda$

where

$N =$ number of activated atoms

$T =$ decay time in min.

$N = N' e^{-\lambda T}$.

4) $N' = Ne^{\lambda T}$

if it is assumed that all radioactivity is due to the irradiation and that N' is the total number of atoms activated at the end of the irradiation.

From equations 1), 2), and 4)

5) $N\lambda e^{\lambda T} = N_s \lambda (1 - e^{-\lambda t})$

or

$N = N_s \, e^{-\lambda T} (1 - e^{-\lambda t})$

During the counting of the foil, the number of atoms which decay will be $N_1 - N_2$ where $N_1 =$ number of radioactive atoms at the start of counting and $N_2 =$ number of radioactive atoms at the end of counting.

$\frac{dN}{d\tau} = N\lambda$

where

$\tau =$ counting time in min.

From similarity to equation 3), it follows that

6) $N_2 = N_1 \, e^{-\lambda \tau}$

7) $N_1 = N_s \, e^{-\lambda T} (1 - e^{-\lambda t})$

from equations 6) and 7)

8) $N_2 = N_s \, e^{-\lambda T} (1 - e^{-\lambda t}) \, e^{-\lambda \tau}$

$(N_1 - N_2) = N_s \, e^{-\lambda T} (1 - e^{-\lambda t}) (1 - e^{-\lambda \tau})$

The counter will not register all the disintegrations because of the geometry and efficiency of the counting assembly and self-absorption in the foil; hence the counts recorded will be

$C = G \, E \, S \, (N_1 - N_2)$

or

$$(N_1 - N_2) = \frac{C}{G\,E\,S}$$

where

G = a geometry factor

E = efficiency

S = self-absorption factor

C = counts.

$$\therefore \quad 9) \quad N_s = \frac{Ce^{\lambda T}}{GES(1 - e^{-\lambda t})\,(1 - e^{-\lambda \tau})}$$

The number of counts will also be a function of the mass of foil material present. This has been taken into account by defining the saturated activity as

$$10) \quad A_s = \frac{N_s \lambda}{m} \quad \text{which becomes}$$

$$11) \quad A_s = \frac{C\lambda\, e^{\lambda T}}{GES\,m(1 - e^{\lambda t})\,(1 - e^{-\lambda \tau})}$$

The values of the constants used in this experiment were:

λ = 0.012836

GE = 0.042

S = 0.150

APPENDIX C

EFFECTIVE CROSS SECTION

From appendix B, equation 11)

1) $$A_s = \frac{C\lambda\, e^{\lambda T}}{G\,E\,S\,m(1 - e^{-\lambda t})\,(1 - e^{-\lambda \tau})}$$

2) $n = n_o\, p\, \sigma_{eff.}/a$

where

$\sigma_{eff.}$ = average atomic cross section

n_o = number of neutrons/min.

n = number of atoms activated/min.

= number of neutrons captured/min.

p = number of target atoms = km

a = target area.

Let

k = number of target atoms/g. = $\frac{\text{Avogadro's number}}{\text{atomic weight}}$

m = mass of target

then

3) $$n = \frac{n_o\, k\, m\, \sigma_{eff.}}{a}$$

4) $$\sigma_{eff.} = \frac{a}{n_o k}\,\frac{n}{m}$$

From equation 2) of appendix B

$$n = N_s\lambda$$

Since the saturated activity is defined in equation 10) of appendix B as

$$A_s = \frac{N_s\lambda}{m}$$

$$\therefore \frac{n}{m} = A_s$$

From equation 4)

5) $$\sigma_{eff.} = \frac{a\, A_s}{n_o k}$$

SUMMARY OF SYMBOLS AND UNITS

Symbols used in report		Units used in report
λ =	decay constant	min.$^{-1}$
$\sigma_{eff.}$ =	effective capture cross section	cm.^{2}
m =	mass	g.
a =	area	cm.^{2}
k =	$\frac{\text{Avogadro's number}}{\text{atomic weight}}$	atoms/g.
A_s =	saturated activity per gram of foil	dis./min./g.
C =	number of counts	
G =	geometry	
E =	efficiency	
W =	weight of foils	g.
p =	number of target atoms	
n =	number of atoms activated/min.	min.$^{-1}$
n_o =	number of neutrons/min.	min.$^{-1}$
N' =	number of atoms activated at time t	
N =	number of radioactive atoms	
N_s =	number of active atoms at saturation	
N_1 =	number of active atoms at start of counting	
N_2 =	number of active atoms at end of counting	
t =	exposure time	min.
τ =	counting time	min.
T =	decay time	min.

EXPERIMENTAL LEAKAGE
NEUTRON SPECTRUM

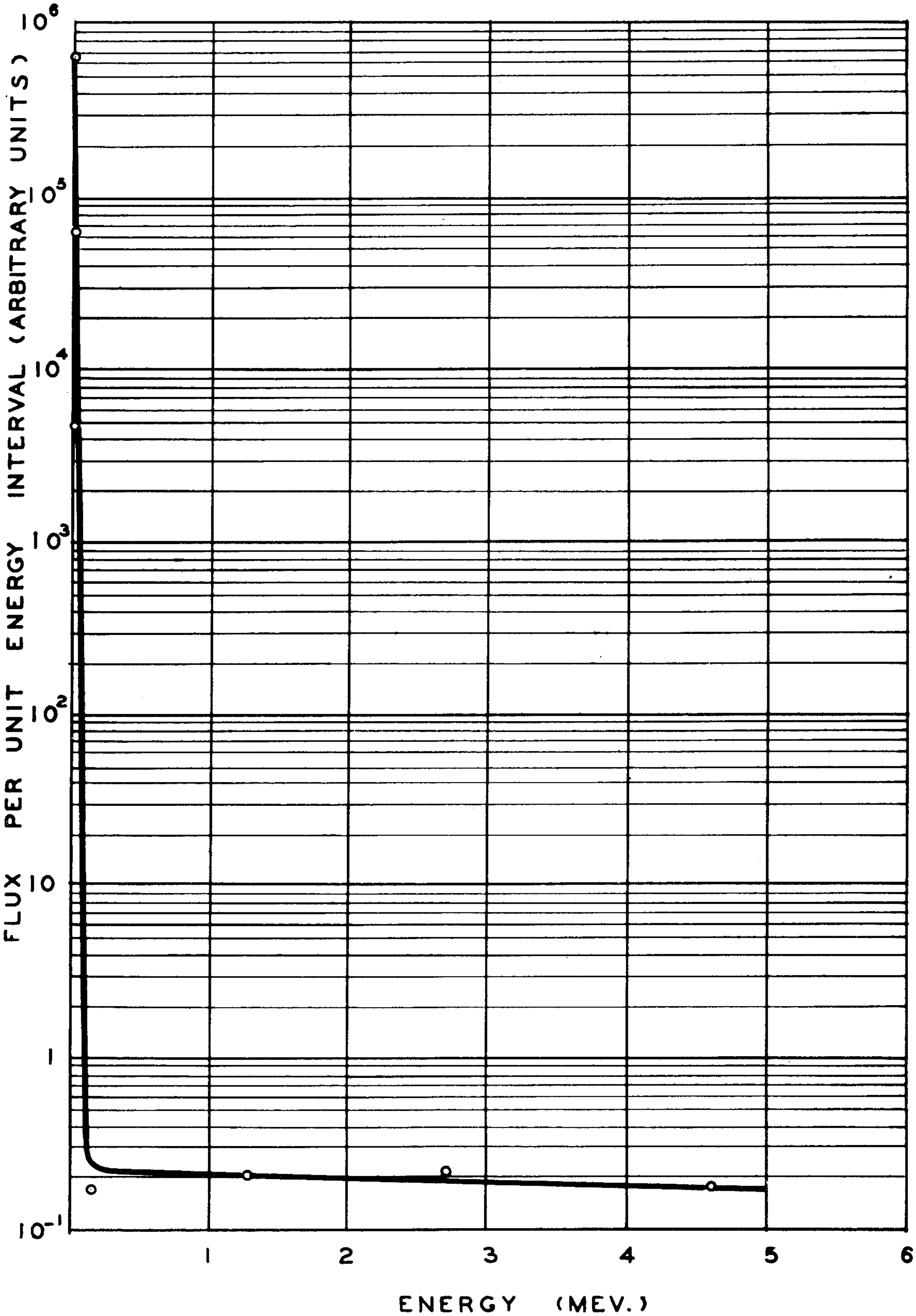
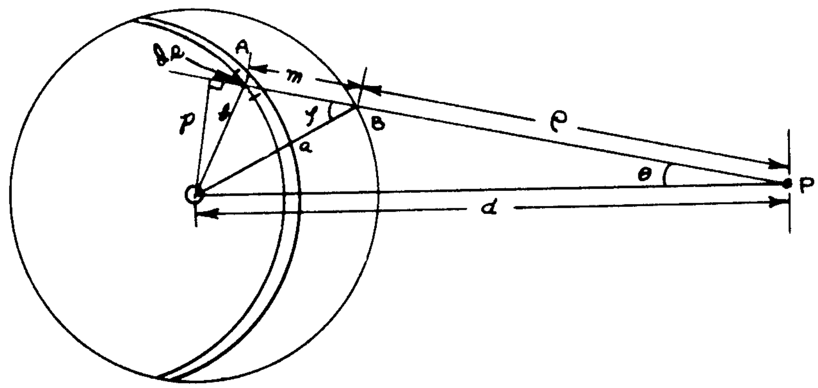


FIG. 1



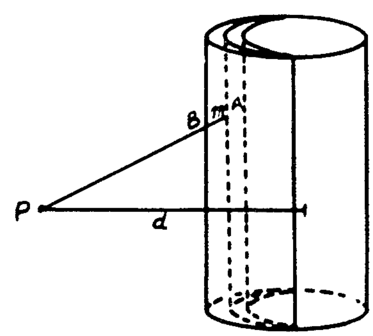


FIG. 2

BARE FOIL ACTIVATION VS DISTANCE FROM CENTER OF REACTOR

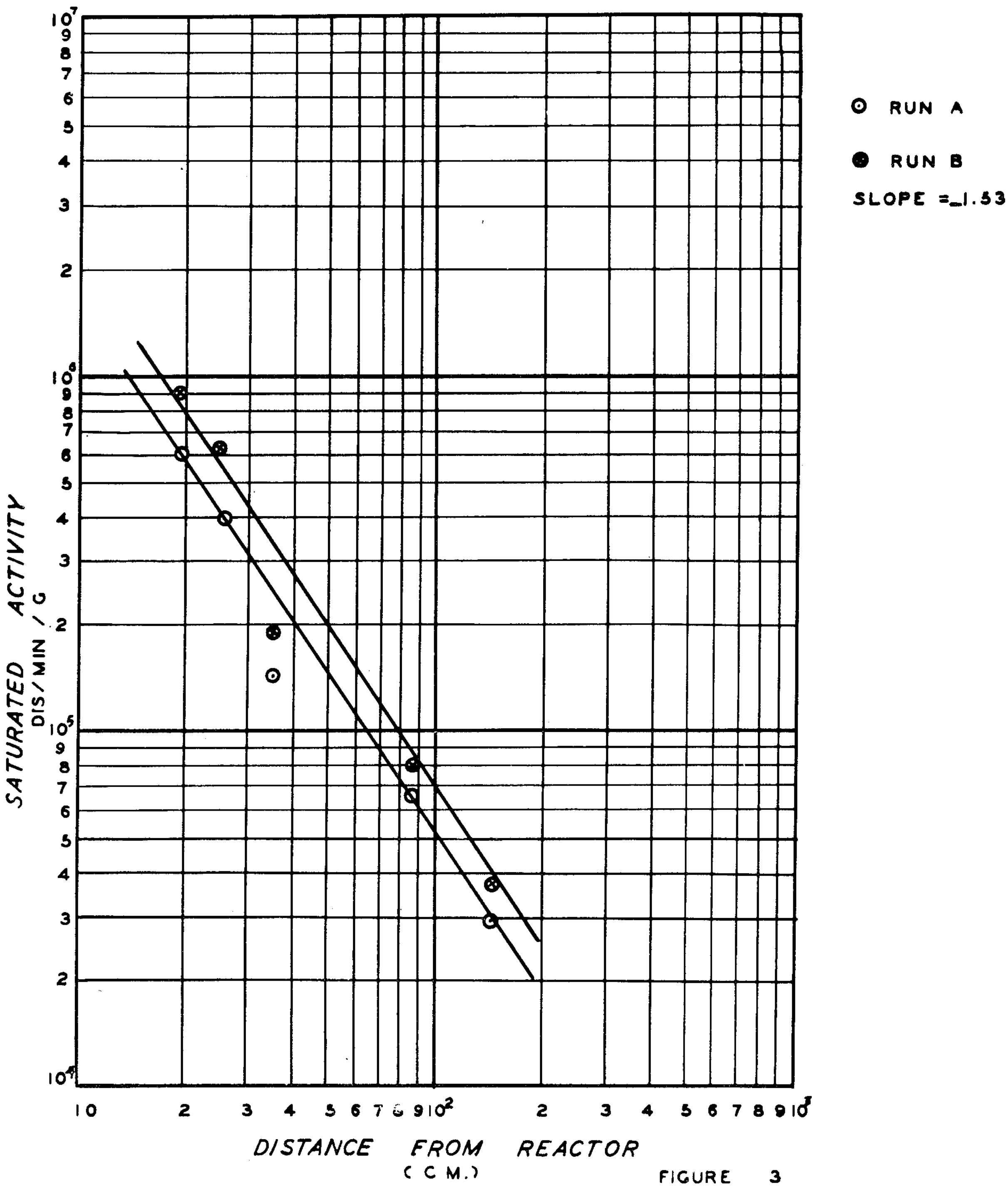


FIGURE 3